

Problem 1.46

[Difficulty: 2]

1.46 The mass of the standard British golf ball is 45.9 ± 0.3 g and its mean diameter is 41.1 ± 0.3 mm. Determine the density and specific gravity of the British golf ball. Estimate the uncertainties in the calculated values.

Given: Standard British golf ball:

$$m = 45.9 \pm 0.3 \text{ g} \quad (20 \text{ to } 1)$$

$$D = 41.1 \pm 0.3 \text{ mm} \quad (20 \text{ to } 1)$$

Find: Density and specific gravity; Estimate of uncertainties in calculated values.

Solution: Density is mass per unit volume, so

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi} \frac{m}{(D/2)^3} = \frac{6}{\pi} \frac{m}{D^3}$$

$$\rho = \frac{6}{\pi} \times 0.0459 \text{ kg} \times \frac{1}{(0.0411)^3} \text{ m}^3 = 1260 \text{ kg/m}^3$$

and

$$SG = \frac{\rho}{\rho_{H_2O}} = 1260 \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{1000 \text{ kg}} = 1.26$$

The uncertainty in density is given by

$$u_\rho = \pm \left[\left(\frac{m}{\rho} \frac{\partial \rho}{\partial m} u_m \right)^2 + \left(\frac{D}{\rho} \frac{\partial \rho}{\partial D} u_D \right)^2 \right]^{\frac{1}{2}}$$

$$\frac{m}{\rho} \frac{\partial \rho}{\partial m} = \frac{m}{\rho} \frac{1}{V} = \frac{V}{V} = 1; \quad u_m = \pm \frac{0.3}{45.9} = \pm 0.654\%$$

$$\frac{D}{\rho} \frac{\partial \rho}{\partial D} = \frac{D}{\rho} \left(-3 \frac{6}{\pi} \frac{m}{D^4} \right) = -3 \left(\frac{6m}{\pi D^4} \right) = -3; \quad u_D = \pm \frac{0.3}{41.1} = \pm 0.730\%$$

Thus

$$u_\rho = \pm \left[u_m^2 + (-3u_D)^2 \right]^{\frac{1}{2}} = \pm \left[0.654^2 + (-3 \times 0.730)^2 \right]^{\frac{1}{2}}$$

$$u_\rho = \pm 2.29\% = \pm 28.9 \text{ kg/m}^3$$

$$u_{SG} = u_\rho = \pm 2.29\% = \pm 0.0289$$

Summarizing

$$\rho = 1260 \pm 28.9 \text{ kg/m}^3 \quad (20 \text{ to } 1)$$

$$SG = 1.26 \pm 0.0289 \quad (20 \text{ to } 1)$$